Trig Derivatives Practice Solutions- 10/26/16

1. Evaluate $\frac{d}{dx}e^x \sin(x)$.

   Solution: Let $f(x) = e^x$ and $g(x) = \sin(x)$, so $f'(x) = e^x$ and $g'(x) = \cos(x)$. Then we can use the product rule to get that $\frac{d}{dx}e^x \sin(x) = e^x \sin(x) + e^x \cos(x) = e^x(\sin(x) + \cos(x))$.

2. Evaluate $\frac{d^2}{dx^2}e^x \sin(x)$.

   Solution: To take the second derivative of $e^x \sin(x)$, we just take the derivative of the derivative, which we already found in problem 1. Thus we want to find $\frac{d}{dx}e^x(\sin(x) + \cos(x))$. Let $f(x) = e^x$ and $g(x) = \sin(x) + \cos(x)$, then $f'(x) = e^x$ and $g'(x) = \cos(x) - \sin(x)$. Then using the product rule, we get $\frac{d^2}{dx^2}e^x \sin(x) = \frac{d}{dx}e^x(\sin(x) + \cos(x)) = e^x(\sin(x) + \cos(x)) + e^x(\cos(x) - \sin(x)) = 2e^x \cos(x)$.

3. Evaluate $\frac{d}{dx}\frac{xe^x}{\cos(x)}$.

   Solution: Let $f(x) = xe^x$ and $g(x) = \cos(x)$. Then to find $f'(x)$, we need to use the product rule, so let $z(x) = x$ and $q(x) = e^x$, so $z'(x) = 1$ and $q'(x) = e^x$. Then using the product rule, we have $f'(x) = e^x + e^x(x) = (x + 1)e^x$. We also know that $g'(x) = -\sin(x)$. Then using the quotient rule, we have that $\frac{d}{dx}\frac{xe^x}{\cos(x)} = \frac{(x+1)e^x \cos(x) - (-\sin(x)xe^x)}{\cos^2(x)} = \frac{e^x((x+1)\cos(x) + x\sin(x))}{\cos^2(x)}$.

4. Evaluate $\frac{d}{dx}\frac{\sin(x)}{\tan(x)}$.

   Solution: We could solve this in two different ways. We could realize that $\frac{\sin(x)}{\tan(x)} = \frac{\sin(x)}{\frac{\sin(x)}{\cos(x)}} = \cos(x)$, so $\frac{d}{dx}\frac{\sin(x)}{\tan(x)} = \frac{d}{dx}\cos(x) = -\sin(x)$. If we didn't realize this, we could use the quotient rule. Let $f(x) = \sin(x)$ and $g(x) = \tan(x)$, so $f'(x) = \cos(x)$ and $g'(x) = \sec^2(x)$. Then $\frac{d}{dx}\frac{\sin(x)}{\tan(x)} = \frac{\cos(x)\tan(x) - \sec^2(x)\sin(x)}{\tan^2(x)} = \frac{\sin(x)(1-\sec^2(x))}{\tan^2(x)} = \frac{(1-\sec^2(x))(\cos(x))}{\tan(x)} = \frac{(1-\sec^2(x))(\cos^2(x))}{\sin(x)} = \frac{\cos^2 - 1}{\sin(x)} = \frac{-\sin^2(x)}{\sin(x)} = -\sin(x)$. Note that we didn't need to simplify that all the way down - I just did that to show that we get the same answer both ways. However, unless we say to simplify completely, it is fine to leave your answer as $\frac{\cos(x)\tan(x) - \sec^2(x)\sin(x)}{\tan^2(x)}$.

5. Evaluate $\frac{d}{dx}x\cot(x)$.

   Solution: Let $f(x) = x$ and $g(x) = \cot(x)$. Then $f'(x) = 1$ and $g'(x) = -\csc^2(x)$ (we did this in class - if you don't remember how we got this, check the review page). Using the product rule, we get $\frac{d}{dx}x\cot(x) = 1 \cdot \cot(x) + (-\csc^2(x))x = \cot(x) - x\csc^2(x)$.

6. Evaluate $\frac{d}{dx}\csc(x)$.

Solution: We can think of this as $\frac{d}{dx}\frac{1}{\sin(x)}$. Let $f(x) = 1$ and $g(x) = \sin(x)$, then $f'(x) = 0$ and $g'(x) = \cos(x)$. Then using the quotient rule, we get $\frac{d}{dx}\csc(x) = \frac{0(\sin(x))-\cos(x)(1)}{\sin^2(x)} = \frac{-\cos(x)}{\sin^2(x)} = -\frac{\cos(x)}{\sin(x)} \cdot \frac{1}{\sin(x)} = -\cot(x)\csc(x)$.